Open quantum system approach to Gibbons-Hawking effect of de Sitter space-time

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Abstract

We analyze, in the paradigm of open quantum systems, the reduced dynamics of a freely falling two-level detector in de Sitter space-time in weak interaction with a reservoir of fluctuating quantized conformal scalar fields in the de Sitter-invariant vacuum. We find that the detector is asymptotically driven to a thermal state at the Gibbons-Hawking temperature, regardless of its initial state. Our discussion therefore shows that the Gibbons-Hawking effect of de Sitter space-time can be understood as a manifestation of thermalization phenomena that involves decoherence and dissipation in open quantum systems.

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The theory of quantum open systems has, in recent years, greatly advanced our understanding of fundamental issues at the foundation of quantum mechanics and nonequilibrium statistical physics, and it has been fruitfully applied to the nascent yet fast-growing fields of quantum information science, modern quantum optics, atomic and many-body systems, soft condensed matter physics, and biophysics. In this Letter, we will apply the open quantum system approach to the investigation of the Gibbons-Hawking effect [1] in the hope of gaining insights into de Sitter space-time from the prospective of quantum open systems. Our interest in this issue also lies in the fact that the AdS/CFT correspondence between quantum gravity on an AdS space-time and a conformal quantum field theory without gravity on a boundary has provided new and rewarding lines of research into many branches of physics, such as condensed matter physics (see Ref. [2] for a recent review) and quantum chromodynamics (see Ref. [3] for a recent review), and there may also exist a holographic duality between quantum gravity on de Sitter space-time and a conformal field theory living on the boundary identified with the timelike infinity of de Sitter space-time [4].

Our plan is to examine the time evolution of a freely falling two-level detector in interaction with fluctuating vacuum conformal scalar fields in de Sitter space-time. The detector is treated as an open quantum system and the vacuum with fluctuations of the quantum fields as the environment. The evolution of the detector is subject to the effects of decoherence and dissipation due to its interaction with the environment, and as for any open system, the full dynamics of the detector can be obtained from the complete time evolution describing the total system (detector plus the environment) by integrating over the field degrees of freedom, which are in fact not observed. We will show that the detector in de Sitter space-time, regardless of its initial state, is asymptotically driven to a thermal state at the Gibbons-Hawking temperature. The open quantum system approach in our Letter therefore demonstrates that the Gibbons-Hawking effect can be understood as a manifestation of thermalization phenomena in the framework of open quantum systems. It is worth noting here that an examination of similar issues, i.e., the Unruh effect and the Hawking radiation, in the paradigm of open quantum systems has been carried out, respectively, in Ref. [5] and Ref. [6]

When vacuum fluctuations are concerned in a curved space-time, one first has to specify the vacuum states. The vacuum states in de Sitter space-time can be classified into two categories: one is the de Sitter-invariant states, the others are those which break de Sitter invariance [7]. Generally, the de Sitter-invariant vacuum, whose status in de Sitter space-time is just like Minkowski vacuum in the flat space-time, is deemed to be a natural vacuum. So we will consider the evolution in the proper time of a freely falling detector in interaction with a quantized conformally coupled massless scalar field in the de Sitter-invariant vacuum.

We assume the combined system (detector + external fluctuating vacuum fields) to be initially prepared in a factorized state, with the detector having two internal energy levels and the fields in the de Sitter vacuum. Thus the detector can be fully described in terms of a two-dimensional Hilbert space, so that its states can be represented by a 2×2 density

matrix ρ, which is Hermitian $\rho^+ = \rho$, and normalized $\mathrm{Tr}(\rho) = 1$ with $\det(\rho) \geq 0$. Without loss of generality, we take the total Hamiltonian for the complete system to have the form

$$H = H_s + H_\phi + \lambda\, H' \; . \tag{1}$$

Here H_s is the Hamiltonian of the detector, which is taken, for simplicity, to be

$$H_s = \frac{\omega_0}{2}\sigma_3 \; , \tag{2}$$

where σ_3 is the Pauli matrix and ω_0 the energy level spacing. H_ϕ is the standard Hamiltonian of conformal scalar fields in de Sitter space-time, details of which need not be specified here and H' is the interaction Hamiltonian of the detector with the external scalar fields and is assumed to be given by

$$H' = \sigma_3 \Phi(x) \; . \tag{3}$$

Let us note here that we can also write H_s and H' in more general forms [5], but that does not change the result of this Letter. In order to achieve a rigorous, mathematically sound derivation of the reduced dynamics of the detector, we will assume that the interaction between the detector and the scalar fields is weak so that the finite-time evolution describing the dynamics of the detector takes the form of a one-parameter semigroup of completely positive maps [8, 9]. It should be pointed out that the coupling constant λ in (1) should be small, and this is required by our assumption that the interaction of the atom with the scalar fields is weak.

Initially, the complete system is described by the total density matrix $\rho_{tot} = \rho(0) \otimes |0\rangle\langle 0|$, where $\rho(0)$ is the initial reduced density matrix of the detector and $|0\rangle$ is the de Sitter-invariant vacuum state for field $\Phi(x)$. In the frame of the detector, the evolution in the proper time τ of the total density matrix ρ_{tot} of the complete system satisfies

$$\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -i L_H[\rho_{tot}(\tau)] \; , \tag{4}$$

where the symbol L_H represents the Liouville operator associated with H

$$L_H[S] \equiv [H, S] \; . \tag{5}$$

The dynamics of the detector can be obtained by tracing over the field degrees of freedom, i.e., by applying the trace projection to the total density matrix $\rho(\tau) = \mathrm{Tr}_\Phi[\rho_{tot}(\tau)]$.

In the limit of weak coupling which we assume in this Letter, the reduced density is found to obey an equation in the Kossakowski-Lindblad form [10, 11]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i\big[H_{\mathrm{eff}},\, \rho(\tau)\big] + \mathcal{L}[\rho(\tau)] \; , \tag{6}$$

where

$$\mathcal{L}[\rho] = \frac{1}{2}\sum_{i,j=1}^{3} a_{ij}\big[2\,\sigma_j \rho\, \sigma_i - \sigma_i\sigma_j\, \rho - \rho\, \sigma_i\sigma_j\big] \; . \tag{7}$$

The matrix a_{ij} and the effective Hamiltonian $H_{\rm eff}$ are determined by the Fourier transform $\mathcal{G}(\lambda)$ and Hilbert transform $\mathcal{K}(\lambda)$ of the field vacuum correlation functions (Wightman functions)

$$G^+(x-y) = \langle 0|\Phi(x)\Phi(y)|0\rangle \;, \tag{8}$$

which are defined as

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} d\tau \, e^{i\lambda\tau}\, G^+\big(x(\tau)\big) \;, \tag{9}$$

$$\mathcal{K}(\lambda) = \frac{P}{\pi i}\int_{-\infty}^{\infty} d\omega \; \frac{\mathcal{G}(\omega)}{\omega-\lambda} \;. \tag{10}$$

Then the coefficients of the Kossakowski matrix a_{ij} can be written explicitly as

$$a_{ij} = A\delta_{ij} - iB\epsilon_{ijk}\delta_{k3} + C\delta_{i3}\delta_{j3} \tag{11}$$

with

$$A = \frac{1}{2}[\mathcal{G}(\omega_0)+\mathcal{G}(-\omega_0)] \;, \quad B = \frac{1}{2}[\mathcal{G}(\omega_0)-\mathcal{G}(-\omega_0)] \;, \quad C = \mathcal{G}(0) - A \;. \tag{12}$$

The effective Hamiltonian $H_{\rm eff}$ contains a correction term, the so-called Lamb shift, and one can show that it can be obtained by replacing ω_0 in H_s with a renormalized energy level spacing Ω as follows

$$H_{\rm eff} = \frac{\Omega}{2}\sigma_3 = \{\omega_0 + i[\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)]\}\sigma_3 \;, \tag{13}$$

where a suitable subtraction is assumed in the definition of $\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)$ to remove the logarithmic divergence which would otherwise be present.

In order to find out how the reduced density evolves with proper time from Eq. (6), we need the Wightman function for the conformally coupled scalar fields in the de Sitter-invariant vacuum. Let us note that there are several different coordinate systems that can be chosen to parametrize de Sitter space-time [12]. Here we choose to work with the global coordinate system (t,χ,θ,ϕ) in which the freely falling detector is comoving with the expansion. The line element is

$$ds^2 = dt^2 - \alpha^2\cosh^2(t/\alpha)[d\chi^2 + \sin\chi^2(d\theta^2 + \sin^2\theta d\varphi^2)] \tag{14}$$

with $\alpha = 3^{1/2}\Lambda^{-1/2}$, where Λ is the cosmological constant. The parameter t is often called the world or cosmic time. The canonical quantization of a massive scalar field with this metric has been done in Ref. [7, 12–20]. In coordinates (14), the wave equation for a massive scalar field becomes

$$\left[\frac{1}{\cosh^3 t/\alpha}\frac{\partial}{\partial t}\left(\cosh^3\frac{t}{\alpha}\,\frac{\partial}{\partial t}\right) - \frac{\Delta}{\alpha^2\cosh^2 t/\alpha} + m^2 + \xi R\right]\phi = 0 \;, \tag{15}$$

where the Laplacian

$$\Delta = \frac{1}{\sin^2\chi}\left[\frac{\partial}{\partial\chi}\left(\sin^2\chi\frac{\partial}{\partial\chi}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] \;. \tag{16}$$

From (15) one can get the eigenmodes, and define a de Sitter-invariant vacuum. Then the Wightman function can be written as [18]

$$G^+(x(\tau), x(\tau'))) = -\frac{1}{16\pi\alpha^2}\,\frac{\frac{1}{4}-\nu^2}{\cos\pi\nu}\,F\left(\frac{3}{2}+\nu, \frac{3}{2}-\nu; 2; \frac{1-Z(x,x')}{2}\right) , \tag{17}$$

where F is a hypergeometric function and

$$\begin{aligned}
&Z(x,x') = \sinh\frac{t}{\alpha}\sinh\frac{t'}{\alpha} - \cosh\frac{t}{\alpha}\cosh\frac{t'}{\alpha}\cos\Omega \\
&\cos\Omega = \cos\chi\cos\chi' + \sin\chi\sin\chi'[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi')]' , \\
&\nu = \left[\frac{9}{4} - \frac{12}{R}(m^2+\xi R)\right]^{1/2} .
\end{aligned} \tag{18}$$

In the massless, conformal coupling limit, for a freely falling detector, the Wightman function becomes

$$G^+(x(\tau), x(\tau'))) = -\frac{1}{16\pi^2\alpha^2\sinh^2(\frac{\tau-\tau'}{2\alpha} - i\varepsilon)} . \tag{19}$$

Its Fourier transform can be found to be

$$\mathcal{G}_{ds}(\lambda) = \frac{\lambda}{2\pi}\,\frac{e^{2\pi\alpha\lambda}}{e^{2\pi\alpha\lambda-1}} . \tag{20}$$

This leads to

$$A = 1/2[\mathcal{G}_{ds}(\lambda) + \mathcal{G}_{ds}(-\lambda)] = \frac{\lambda\coth(\pi\alpha\lambda)}{4\pi} , \quad B = 1/2[\mathcal{G}_{ds}(\lambda) - \mathcal{G}_{ds}(-\lambda)] = \frac{\lambda}{4\pi} . \tag{21}$$

In order to solve Eq.(6) to find out how the reduced density evolves with proper time, let us express it in terms of the Pauli matrices,

$$\rho(\tau) = \frac{1}{2}\left(1 + \sum_{i=1}^{3}\rho_i(\tau)\sigma_i\right) . \tag{22}$$

Substituting Eq. (22) into Eq. (6), one can show that the Bloch vector $|\rho(\tau)\rangle$ of components $\{\rho_1(\tau), \rho_2(\tau), \rho_3(\tau)\}$ obeys

$$\frac{\partial}{\partial\tau}|\rho(\tau)\rangle = -2\mathcal{H}|\rho(\tau)\rangle + |\eta\rangle , \tag{23}$$

where $|\eta\rangle$ denotes a constant vector $\{0, 0, -4B\}$. The exact form of the matrix $\mathcal{H}$ reads

$$\mathcal{H} = \begin{pmatrix} 2A+C & \Omega/2 & 0 \\ -\Omega/2 & 2A+C & 0 \\ 0 & 0 & 2A \end{pmatrix} . \tag{24}$$

This matrix is nonsingular and its three eigenvalues are $\lambda_1 = 2A$, $\lambda_\pm = (2A+C) \pm i\Omega/2$. Since the real parts of these eigenvalues are positive, at later times, $|\rho(\tau)\rangle$ will reach an equilibrium state $|\rho(\infty)\rangle$ [21], which can be found by formally solving Eq. (23)

$$|\rho(\tau)\rangle = e^{-2\mathcal{H}\tau}|\rho(0)\rangle + (1 - e^{-2\mathcal{H}\tau})|\rho_\infty\rangle , \tag{25}$$

with

$$|\rho_\infty\rangle = \frac{1}{2}\mathcal{H}^{-1}|\eta\rangle = -\frac{B}{A}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} . \tag{26}$$

If we let $\beta = 1/T = 2\ \mathrm{arctanh}(B/A)/\omega_0$, we can easily show that Eq. (26) can be rewritten in a purely thermal form

$$\rho_\infty = \frac{e^{-\beta H_s}}{\mathrm{Tr}[e^{-\beta H_s}]} . \tag{27}$$

Making use of Eq. (21), we find the temperature of the thermal state is

$$T = \frac{\omega}{2\mathrm{arctanh}(B/A)} = \frac{1}{2\pi\alpha} . \tag{28}$$

This is exactly the Gibbons-Hawking temperature. Thus, regardless of its initial state, a freely falling two-level detector in de Sitter space-time is asymptotically driven to a thermal state at the Gibbons-Hawking temperature. Therefore, there must exist a bath of thermal radiation in de Sitter space-time. Our open system approach thus reveals that the existence of the Gibbons-Hawking effect is simply a manifestation of thermalization phenomena in the framework of open system dynamics. At this point, it is worth noting that the Gibbons-Hawking temperature of de Sitter space-time has also be derived in other different contexts such as the global embedding approach [22, 23] and the universal Rindler scheme [24].

Further aspects of the Gibbons-Hawking effect in terms of the thermalization phenomena can be studied by examining the behavior of the finite-time solution (25). For this purpose, let us note that

$$e^{-2\mathcal{H}\tau} = \frac{4}{\Omega^2+4C^2}\left\{e^{-4A\tau}\Lambda_1 + 2e^{-2(2A+C)\tau}\left[\Lambda_2\cos(\Omega\tau) + \Lambda_3\frac{\sin(\Omega\tau)}{\Omega}\right]\right\} , \tag{29}$$

where

$$\begin{aligned}
\Lambda_1 &= \big[(2A+C)^2 + \frac{\Omega^2}{4}\big]I - 2(2A+C)\mathcal{H} + \mathcal{H}^2 , \\
\Lambda_2 &= -2A(A+C)I + (2A+C)\mathcal{H} - \frac{1}{2}\mathcal{H}^2 , \\
\Lambda_3 &= 2A\big[\frac{\Omega^2}{4} - C(2A+C)\big]I + \big[C(4A+C) - \frac{\Omega^2}{4}\big]\mathcal{H} - C\mathcal{H}^2 .
\end{aligned} \tag{30}$$

Equation (29) reveals that a freely falling detector in de Sitter space-time is subjected to the effects of decoherence and dissipation characterized by the exponentially decaying factors involving the real parts of the eigenvalues of $\mathcal{H}$ and oscillating terms associated with the imaginary part. Therefore, the Gibbons-Hawking effect as a manifestation of thermalization phenomena in the framework of open quantum systems actually involves phenomena of decoherence and dissipation. This suggests that the vacuum in de Sitter space-time behaves like a fluctuating as well as a dissipative medium [25, 26]. In this regard, our approach to the

derivation of the Gibbons-Hawking effect seems to shed new light on the issue as compared to other traditional treatments, and it ties its existence to the effects of decoherence and dissipation in open quantum systems .

In summary, we have analyzed, using the well-known techniques in the study of open quantum systems, the time evolution of a freely falling detector in de Sitter space-time in weak interaction with fluctuating vacuum conformal scalar fields in the de Sitter-invariant vacuum. The detector has been shown to be asymptotically driven to a thermal state at the Gibbons-Hawking temperature, regardless of its initial state. Our open system approach to the issue therefore shows that the Gibbons-Hawking effect of de Sitter space-time can be understood as a manifestation of thermalization phenomena in the framework of open quantum systems, which actually involves the effects of decoherence and dissipation. It is worthwhile to note that the general techniques developed in the theory of open quantum systems may also be applicable to studying other phenomena in curved space-times, such as particle creation, and may thus provide new insights in the physical understanding of these phenomena.

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